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CASE METHOD EMPLOYED FOR SOLVING THE PROBLEM OF THERMAL CREEP OF A RAREFIED GAS ALONG A SOLID CYLINDRICAL SURFACE

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An analytical solution of a half-space boundary-value problem is constructed for an inhomogeneous kinetic Boltzmann equation with the collision operator in the form of an operator of an ellipsoidal statistical model in the problem on thermal creep of a rarefied gas along a solid cylindrical surface. Corrections to the thermal creep coefficient are obtained for the cases of longitudinal and transverse flow past a straight circular cylinder in the approximation linear with respect to the Knudsen number, allowing for the interfacial curvature. The results are compared with available data.

Introduction. At present, it is quite common to apply the method of elementary solutions (Case method) [1] to homogeneous model kinetic equations. However, to the best of our knowledge, only in a few cases has this method been applied to solve inhomogeneous kinetic equations. Thus, from the solution of an inhomogeneous linearized kinetic Boltzmann equation with a collision operator in the form of the BGK (Bhatnagar–Gross–Krook) model operator in the Knudsen layer, Akimov and Gaidukov [2] determined the creep velocity of a rarefied gas along a solid spherical surface. At the same time, in the approximation linear in terms of the Knudsen number, accurate analytic expressions in a closed form were obtained for the corrections to the coefficients of thermal and isothermal creep with allowance for interfacial curvature. No numerical analysis has been performed, though, because the resultant expressions were rather complex. In [3], the solution of the problem on second-order thermal creep was constructed using the kinetic Boltzmann equation with the collision operator in the form of the BGK operator. For the same reason as above, no numerical analysis of the results obtained was carried out. In [4], the Case method was applied to the inhomogeneous linearized kinetic Boltzmann equation with the collision operator in the form of an operator of the ellipsoid statistical model in the problem of rarefied gas creep along a solid spherical surface. The value of β_R, a coefficient allowing for the thermal creep coefficient versus the interfacial curvature radius, was found by the numerical analysis of the obtained analytical expressions.

In this study, the problem of rarefied gas thermal creep along the surface of a straight circular cylinder is solved by the Case method. Unlike [4], some integrals entering into the expression for the coefficient β_R are analytically calculated, and the final result is expressed in terms of Loyalka integrals [5]. The results obtained are required to calculate the thermophoresis rate of cylindrical aerosol particles [6].

1. Cross-Flow Past a Cylinder. Derivation of Governing Equations. Let us consider a rarefied gas flow inhomogeneous in temperature past a solid cylindrical surface with small deviations from the equilibrium state. The gas flow will be described by the Boltzmann equation with a linearized collision operator in the form of the operator of the ellipsoidal statistical model [7, 8]. In a cylindrical system of coordinates where the Oz axis coincides with the cylinder axis, the equation for the considered model is written in the following form:

$$C_\rho \frac{\partial f}{\partial \rho} + f(\rho,\varphi,\boldsymbol{C}) + \frac{1}{\rho}\Big(C_\varphi^2 \frac{\partial f}{\partial C_\rho} - C_\rho C_\varphi \frac{\partial f}{\partial C_\varphi} + C_\varphi \frac{\partial f}{\partial \varphi}\Big) = f^0(\boldsymbol{C})\bigg(1 + \beta^{-3/2}\iiint K(\boldsymbol{C},\boldsymbol{C}')f(\rho,\varphi,\boldsymbol{C}')\,d\boldsymbol{C}'\bigg).$$

Here $f(\rho,\varphi,\boldsymbol{C})$ is the distribution function of gas molecules in coordinates and velocities, $f^0(\boldsymbol{C}) = (\beta/\pi)^{3/2}\exp(-C^2)$ is an absolute Maxwellian, $\beta = m/(2k_{\rm B}T)$, $\rho(3\mu_{\rm g}/(2p))\beta^{-1/2}$ is the distance counted from

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the cylinder axis, ρ is a dimensionless coordinate, $\beta^{-1/2}C_i$ are the components of the gas molecule eigenvelocity, $\mu_{\rm g}$ is the dynamic viscosity of the gas, p is the static pressure, $k_{\rm B}$ is the Boltzmann constant, m is the mass of gas particles, and $K(\boldsymbol{C},\boldsymbol{C}') = 1 + 2\boldsymbol{C}\boldsymbol{C}' + 2(C^2 - 3/2)(C'^2 - 3/2)/3 - 2C_iC_j(C_i'C_j' - \delta_{ij}\boldsymbol{C}'^2/3)$.

The condition of diffuse reflection is assumed as the boundary condition on the wall.

Let us take that the temperature gradient at a distance from the cylindrical surface is perpendicular to its axis. Let us linearize the distribution function that describes the gas state in a locally equilibrium distribution function in the Chapman–Enskog approximation [9], i.e., let us represent it as

$$f(\rho,\varphi,\boldsymbol{C}) = f^0(\boldsymbol{C})[1 + Y(\rho,\varphi,\boldsymbol{C})].$$

Let us expand the function $Y(\rho,\varphi,\boldsymbol{C})$, allowing for the deviations of velocity and coordinate distributions of gas molecules in the Knudsen layer from the distribution function in the gas volume as a power series in a small parameter $1/R$:

$$Y(\rho,\varphi,\boldsymbol{C}) = Y^{(1)}(\rho,\varphi,\boldsymbol{C}) + R^{-1}Y^{(2)}(\rho,\varphi,\boldsymbol{C}) + \ldots\ . \tag{1.1}$$

By this expansion, we obtain a system of one-dimensional integrodifferential equations

$$C_\rho \frac{\partial Y^{(1)}}{\partial\rho} + Y^{(1)}(\rho,\varphi,\boldsymbol{C}) = \pi^{-3/2}\int \exp(-C'^2)\, K(\boldsymbol{C},\boldsymbol{C}')Y^{(1)}(\rho,\varphi,\boldsymbol{C}')\, d\boldsymbol{C}'; \tag{1.2}$$

$$C_\rho \frac{\partial Y^{(2)}}{\partial\rho} + Y^{(2)}(\rho,\varphi,\boldsymbol{C}) = \pi^{-3/2}\int \exp(-C'^2)\, K(\boldsymbol{C},\boldsymbol{C}')Y^{(2)}(\rho,\varphi,\boldsymbol{C}')\, d\boldsymbol{C}'$$

$$- C_\varphi^2 \frac{\partial Y^{(1)}}{\partial C_\rho} + C_\rho C_\varphi \frac{\partial Y^{(1)}}{\partial C_\varphi} - C_\varphi \frac{\partial Y^{(1)}}{\partial\varphi} \tag{1.3}$$

with the boundary conditions

$$Y^{(1)}(R,\varphi,\boldsymbol{C}) = -2C_\varphi U_\varphi^{(1)}\Big|_S + C_\varphi(C^2 - 5/2)k, \qquad C_\rho > 0,$$

$$Y^{(2)}(R,\varphi,\boldsymbol{C}) = -2C_\varphi U_\varphi^{(2)}\Big|_S, \qquad C_\rho > 0,$$

$$Y^{(1)}(\infty,\varphi,\boldsymbol{C}) = 0, \qquad Y^{(2)}(\infty,\varphi,\boldsymbol{C}) = 0,$$

from which we find an expression for the two first terms of expansion (1.1). Here $3R\mu_{\rm g}\beta^{-1/2}/(2p)$ is the cylinder radius, S is the cylindrical surface, $k = \dfrac{1}{T_S}\dfrac{\partial T}{R\,\partial\varphi}\Big|_S$, $\beta^{-1/2}U_i$ are the components of the mass-averaged velocity of the flow. Equation (1.2) describes the processes on the boundary of a solid flat surface, and equation (1.3) makes it possible to allow for the influence of the interfacial curvature.

The solution of (1.2) is sought as the expansion in terms of two orthogonal polynomials

$$Y^{(1)}(\rho,\varphi,\boldsymbol{C}) = C_\varphi Y_1^{(1)}(\rho,\varphi,C_\rho) + C_\varphi(C_\varphi^2 + C_z^2 - 2)Y_2^{(1)}(\rho,\varphi,C_\rho). \tag{1.4}$$

Note that orthogonality in (1.4) is understood as the scalar product

$$(f,g) = \int\limits_{-\infty}^{+\infty} f(\rho,\varphi,\boldsymbol{C})g(\rho,\varphi,\boldsymbol{C})\exp(-C^2)\, d^3\boldsymbol{C}.$$

The solution of (1.3) is sought in the following form:

$$Y^{(2)}(\rho,\varphi,\boldsymbol{C}) = C_\varphi Y_1^{(2)}(\rho,\varphi,C_\rho). \tag{1.5}$$

Let us denote $\mu = C_\rho$. Then, by substituting expansions (1.4) and (1.5) into (1.3), multiplying the resultant relation by $C_\varphi \exp(-C_\varphi^2 - C_z^2)$, and integrating over C_φ and C_z from $-\infty$ to $+\infty$, we obtain the following equation for the function $Y_1^{(2)}(\rho,\varphi,\mu)$:

$$\mu \frac{\partial Y_1^{(2)}}{\partial \rho} + Y_1^{(2)}(\rho, \varphi, \mu) = \frac{1}{\sqrt{\pi}} \int\limits_{-\infty}^{\infty} Y_1^{(2)}(\rho, \varphi, \mu') \exp(-\mu'^2)\, d\mu'$$

$$- \frac{\mu}{\sqrt{\pi}} \int\limits_{-\infty}^{\infty} \mu' Y_1^{(2)}(\rho, \varphi, \mu') \exp(-\mu'^2)\, d\mu' + \mu Y_1^{(1)}(\rho, \varphi, \mu) - \frac{3}{2} \frac{\partial Y_1^{(1)}}{\partial \mu} + 3\mu Y_2^{(1)}(\rho, \varphi, \mu) - \frac{3}{2} \frac{\partial Y_2^{(1)}}{\partial \mu} \tag{1.6}$$

with the boundary conditions

$$Y_1^{(2)}(R, \varphi, \mu) = -2U_\varphi^{(2)}\Big|_S, \quad \mu > 0, \qquad Y_1^{(2)}(\infty, \varphi, \mu) = 0. \tag{1.7}$$

Here $Y_1^{(1)}(\rho, \varphi, \mu)$ and $Y_2^{(1)}(\rho, \varphi, \mu)$ are the distribution functions from the problem of rarefied gas thermal creep along a solid flat surface [8]:

$$Y_1^{(1)}(\rho, \varphi, \mu) = \int\limits_0^{\infty} a(\eta, \varphi)\, F(\eta, \mu) \exp\Big(-\frac{x}{\eta}\Big)\, d\eta, \qquad x = \rho - R; \tag{1.8}$$

$$Y_2^{(1)}(\rho, \varphi, \mu) = k \int\limits_0^{\infty} \exp\Big(-\frac{x}{\eta}\Big) \delta(\eta - \mu)\, d\eta, \tag{1.9}$$

$$F(\eta, \mu) = \frac{1}{\sqrt{\pi}}\, \eta P \frac{1}{\eta - \mu} + \exp(\eta^2) \lambda(\eta) \delta(\eta - \mu), \quad \lambda(z) = 1 + \frac{1}{\sqrt{\pi}}\, z \int\limits_{-\infty}^{\infty} \frac{\exp(-\mu^2)}{\mu - z}\, d\mu;$$

$$a(\eta, \varphi) = \eta(\eta - Q_1) \exp(-\eta^2) X(-\eta) k / (2|\lambda^+(\eta)|^2), \quad \lambda^{\pm}(\eta) = \lambda(\eta) \pm \sqrt{\pi}\, i\eta \exp(-\eta^2),$$
$$X(z) = \frac{1}{z} \exp\left(\frac{1}{\pi} \int\limits_0^{\infty} \frac{\theta(\tau) - \pi}{\tau - z}\, d\tau\right), \quad \theta(\tau) - \pi = -\frac{\pi}{2} - \operatorname{arctg} \frac{\lambda(\tau)}{\sqrt{\pi}\tau \exp(-\tau^2)}, \tag{1.10}$$

$\lambda(z)$ is the Cercignani dispersion function, Px^{-1} is the distribution in terms of the principal integral value in integration x^{-1}, $\delta(x)$ is the Dirac delta-function, $\theta(\tau)$ is the single-valued regular branch of the argument of the function $\lambda^+(\tau)$, satisfying the condition $\theta(0) = 0$.

Therefore, the problem is reduced to the solution of Eq. (1.6) with the boundary conditions (1.7).

2. Allowance for the Interfacial Curvature Influence on the Thermal Creep Coefficient. The solution of (1.6) is sought in the form

$$Y_1^{(2)}(\rho, \varphi, \mu) = \int\limits_0^{\infty} \psi(\eta, \varphi, \mu) \exp\Big(-\frac{x}{\eta}\Big)\, d\eta. \tag{2.1}$$

By substituting (1.8), (1.9), and (2.1) in (1.6), we obtain an inhomogeneous characteristic equation

$$\Big(1 - \frac{\mu}{\eta}\Big) \psi(\eta, \varphi, \mu) = \frac{1}{\sqrt{\pi}} \int\limits_{-\infty}^{\infty} \psi(\eta, \varphi, \mu') \exp(-\mu'^2)\, d\mu'$$

$$- \frac{1}{\sqrt{\pi}}\, \mu \int\limits_{-\infty}^{\infty} \mu' \psi(\eta, \varphi, \mu') \exp(-\mu'^2)\, d\mu' + Z(\eta, \varphi, \mu); \tag{2.2}$$

$$Z(\eta, \varphi, \mu) = \mu a(\eta, \varphi)\, F(\eta, \mu) - \frac{3}{2}\, a(\eta, \varphi) \frac{\partial F}{\partial \mu} + 3\mu k \delta(\eta - \mu) - \frac{3k}{2} \frac{\partial}{\partial \mu} \delta(\eta - \mu). \tag{2.3}$$

By multiplying (2.2) by $\exp(-\mu^2)$ and integrating over μ from $-\infty$ to ∞, we find

$$\int\limits_{-\infty}^{\infty} \mu \psi(\eta, \varphi, \mu) \exp(-\mu^2)\, d\mu = -\eta \int\limits_{-\infty}^{\infty} Z(\eta, \varphi, \mu) \exp(-\mu^2)\, d\mu.$$

Taking into account that the value of the last integral equals zero [4], we write (2.2) in the following form:

$$(\eta-\mu)\psi(\eta,\varphi,\mu)=\eta m(\eta,\varphi)/\sqrt{\pi}+\eta Z(\eta,\varphi,\mu); \tag{2.4}$$

$$m(\eta,\varphi)=\int\limits_{-\infty}^{\infty}\psi(\eta,\varphi,\mu)\exp(-\mu^2)\,d\mu. \tag{2.5}$$

The general solution of Eq. (2.4) in the space of generalized functions has the form [10]

$$\psi(\eta,\varphi,\mu)=\eta P/(\eta-\mu)(m(\eta,\varphi)/\sqrt{\pi}+Z(\eta,\varphi,\mu))+g(\eta,\varphi)\delta(\eta-\mu). \tag{2.6}$$

The explicit form of the function $g(\eta,\varphi)$ is found by substituting (2.6) into (2.5):

$$g(\eta,\varphi)=\left(m(\eta,\varphi)\lambda(\eta)-\eta\int\limits_{-\infty}^{\infty}P\,\frac{1}{\eta-\mu}\,Z(\eta,\varphi,\mu)\exp(-\mu^2)\,d\mu\right)\exp(\eta^2).$$

It was shown in [4] that

$$\int\limits_{-\infty}^{\infty}P\,\frac{1}{\eta-\mu}\,\mu F(\eta,\mu)\exp(-\mu^2)\,d\mu=-1,\qquad \int\limits_{-\infty}^{\infty}P\,\frac{1}{\eta-\mu}\Big(F(\eta,\mu)\Big)'_{\mu}\exp(-\mu^2)\,d\mu=-1,$$

$$\int\limits_{-\infty}^{\infty}P\,\frac{1}{\eta-\mu}\,\mu\delta(\eta-\mu)\exp(-\mu^2)\,d\mu=2\exp(-\eta^2)\Big(\eta^2-\frac{1}{2}\Big),$$

$$\int\limits_{-\infty}^{\infty}P\,\frac{1}{\eta-\mu}\Big(\delta(\eta-\mu)\Big)'_{\mu}\exp(-\mu^2)\,d\mu=2\exp(-\eta^2)\Big(\eta^2-\frac{1}{2}\Big).$$

Hence, with allowance for (2.3), we have

$$\int\limits_{-\infty}^{\infty}P\,\frac{1}{\eta-\mu}\,Z(\eta,\varphi,\mu)\exp(-\mu^2)\,d\mu=\frac{1}{2}\,a(\eta,\varphi)+3k\exp(-\eta^2)\Big(\eta^2-\frac{1}{2}\Big).$$

Thus,

$$\psi(\eta,\varphi,\mu)=\eta P/(\eta-\mu)[m(\eta,\varphi)/\sqrt{\pi}+Z(\eta,\varphi,\mu)]$$
$$+[m(\eta,\varphi)\exp(\eta^2)\lambda(\eta)-\eta a(\eta,\varphi)\exp(\eta^2)/2-3k\eta(\eta^2-1/2)]\delta(\eta-\mu). \tag{2.7}$$

The solution of (2.1) automatically satisfies the boundary condition (1.7) at infinity. By substituting (2.7) into (2.1) and allowing for the boundary condition (1.7) at the cylindrical surface, we obtain a singular integral equation with a kernel of the Cauchy type [11]

$$-2U_{\varphi}^{(2)}\Big|_{S}=\frac{1}{\sqrt{\pi}}\int\limits_{0}^{\infty}\frac{\eta m(\eta,\varphi)}{\eta-\mu}\,d\eta+\int\limits_{0}^{\infty}\eta Z(\eta,\varphi,\mu)\,\frac{d\eta}{\eta-\mu}$$
$$+m(\mu,\varphi)\exp(\mu^2)\lambda(\mu)-\mu a(\mu,\varphi)\exp(\mu^2)/2-3k\mu(\mu^2-1/2),\quad \mu>0. \tag{2.8}$$

It was shown in [4] that

$$\int\limits_{0}^{\infty}\eta P\,\frac{1}{\eta-\mu}\,a(\eta,\varphi)\,F(\eta,\mu)\,d\eta=\Big(\mu\,Y_1^{(1)}(R,\varphi,\mu)\Big)'_{\mu},$$

$$\int\limits_{0}^{\infty}\eta P\,\frac{1}{\eta-\mu}\Big(a(\eta,\varphi)F(\eta,\mu)\Big)'_{\mu}\,d\eta=\frac{1}{2}\Big(\mu Y_1^{(1)}(R,\varphi,\mu)\Big)''_{\mu\mu},$$

$$\int_0^\infty \eta P \frac{1}{\eta-\mu}\,\delta(\eta-\mu)\,d\eta = 1, \qquad \int_0^\infty \eta P \frac{1}{\eta-\mu}\,\Bigl(\delta(\eta-\mu)\Bigr)'_\mu\,d\eta = 0.$$

Then, taking into consideration that $Y_1^{(1)}(R,\varphi,\eta) = (\eta^2+Q_2)k$ [8], we find

$$\int_0^\infty \eta Z(\eta,\varphi,\mu)\,\frac{d\eta}{\eta-\mu} = \Bigl(3\mu^3 + Q_2\mu - \frac{3}{2}\,\mu\Bigr)k.$$

Here Q_n are the Loyalka integrals [5]:

$$Q_n = \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{t^{n+1}\exp(-t^2)\,dt}{X(-t)}.$$

With allowance for the results obtained, we write (2.8) in the following form:

$$f(\mu,\varphi) = m(\mu,\varphi)\exp(\mu^2)\lambda(\mu) + \frac{1}{\sqrt{\pi}}\int_0^\infty \frac{\eta m(\eta,\varphi)}{\eta-\mu}\,d\eta, \qquad \mu > 0; \tag{2.9}$$

$$f(\mu,\varphi) = -2U_\varphi^{(2)}|_S - Q_2\mu + a(\eta,\varphi)\exp(\eta^2)/2. \tag{2.10}$$

Let us introduce an auxiliary function

$$M(z,\varphi) = \frac{1}{2\pi i}\int_0^\infty \frac{\eta m(\eta,\varphi)}{\eta-z}\,d\eta.$$

Allowing for the boundary values of the functions $M(z,\varphi)$ and $\lambda(z)$ on the upper and lower flanks of cuts ($[0,\infty)$ and $(-\infty,+\infty)$, respectively), (2.9) is reduced to a half-space boundary-value Riemann problem [11]

$$M^+(\mu,\varphi)\lambda^+(\mu) - M^-(\mu,\varphi)\lambda^-(\mu) = \mu f(\mu,\varphi)\exp(-\mu^2), \quad \mu > 0. \tag{2.11}$$

The coefficient of the boundary-value problem (2.11) coincides with the coefficient of the boundary-value problem on gas creep along a solid flat surface [8]. With allowance for this, (2.11) is reduced to the discontinuity problem [11]

$$M^+(\mu,\varphi)X^+(\mu) - M^-(\mu,\varphi)X^-(\mu) = \mu f(\mu,\varphi)\exp(-\mu^2)X^-(\mu)/\lambda^-(\mu), \qquad \mu > 0,$$

which has a solution vanishing at infinity when the following condition [8] is fulfilled:

$$\frac{2}{\sqrt{\pi}}\int_0^\infty \frac{f(t,\varphi)}{X(-t)}\,t\exp(-t^2)\,dt = 0. \tag{2.12}$$

By substituting (2.10) into (2.12), with allowance for (1.10), we obtain

$$U_\varphi^{(2)}\Big|_S = \frac{k}{2}\Biggl(Q_1Q_2 - \frac{1}{2\sqrt{\pi}}\int_0^\infty \frac{t^2(t-Q_1)}{|\lambda^+(t)|^2}\exp(-t^2)\,dt\Biggr).$$

Since

$$\frac{1}{\sqrt{\pi}}\int_0^\infty \frac{t^2(t-Q_1)}{|\lambda^+(t)|^2}\exp(-t^2)\,dt = -3Q_3 - Q_1Q_2, \tag{2.13}$$

then $U_\varphi^{(2)}|_S = 3k(Q_3+Q_1Q_2)/4$. By substituting the values of Loyalka integrals $Q_1 = -1.01619$, $Q_2 = -1.26663$, $Q_3 = -1.8207$ into the expression obtained [5], we have $U_\varphi^{(2)}\Big|_S = -0.40017k$. From this, allowing for (1.1), we find the velocity of rarefied gas thermal creep along a solid cylindrical surface

$$U_\varphi\Big|_S = U_\varphi^{(1)}\Big|_S + R^{-1}U_\varphi^{(2)}\Big|_S = (0.38332 - 0.400\,17R^{-1})k. \tag{2.14}$$

Taking into account the relation $\lambda = \nu(\pi\beta)^{1/2}$ between the kinematic viscosity ν of the gas and the mean free path of gas particles λ and using the conventional method of normalization of physical quantities, we find

$$\frac{1}{R} = \frac{3\mu_{\mathrm{g}}}{2p\beta^{1/2}R^*} = \frac{3\nu\beta^{1/2}}{R^*} = \frac{3}{\sqrt{\pi}}\,\frac{\lambda}{R^*} = \frac{3}{\sqrt{\pi}}\,\mathrm{Kn}.$$

Converting (2.14) to dimensional quantities, we obtain

$$U_\varphi\Big|_S = 1.149\,95\nu\,(1 - 1.7684\,\mathrm{Kn})\,\frac{1}{T_S}\,\frac{\partial T}{R^*\,\partial\varphi}\Big|_S.$$

Here R^* is the dimensional radius of the cylinder and $\mathrm{Kn} = \lambda/R^*$ is the Knudsen number.

Therefore, in the case of a rarefied gas flow past a solid cylindrical surface, we have $\beta_{R\perp} = 1.7684$.

3. Longitudinal Flow Past a Cylindrical Surface. Let us assume that the temperature gradient far from the cylindrical surface is directed along its axis. Let us use the notation $k_1 = (1/T_S)\partial T/\partial z\Big|_S$.

The solution of (1.2) and (1.3) is sought in the following form:

$$Y^{(1)}(\rho, \boldsymbol{C}) = C_z Y_1^{(1)}(\rho, C_\rho) + C_z(C_\varphi^2 + C_z^2 - 2)Y_2^{(1)}(\rho, C_\rho), \quad Y^{(2)}(\rho, \boldsymbol{C}) = C_z Y_1^{(2)}(\rho, C_\rho). \tag{3.1}$$

By substituting expansions (3.1) into (1.3), multiplying the resultant expression by $C_z \exp{(-C_\varphi^2 - C_z^2)}$, and integrating over C_φ and C_z from $-\infty$ to $+\infty$, we obtain the following equation for the function $Y_1^{(2)}(\rho, \mu)$:

$$\mu\,\frac{\partial Y_1^{(2)}}{\partial\rho} + Y_1^{(2)}(\rho, \mu) = \frac{1}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty} Y_1^{(2)}(\rho, \mu')\exp{(-\mu'^2)}\,d\mu'$$

$$-\frac{\mu}{\sqrt{\pi}}\int\limits_{-\infty}^{\infty}\mu' Y_1^{(2)}(\rho, \mu')\,\exp{(-\mu'^2)}\,d\mu' - \frac{1}{2}\,\frac{\partial Y_1^{(1)}}{\partial\mu} + \mu Y_2^{(1)}(\rho, \mu) - \frac{1}{2}\,\frac{\partial Y_2^{(1)}}{\partial\mu}$$

with the boundary conditions

$$Y_1^{(2)}(R, \mu) = -2U_z^{(2)}\Big|_S, \quad \mu > 0, \qquad Y_1^{(2)}(\infty, \mu) = 0.$$

Here

$$Y_1^{(1)}(\rho, \mu) = \int\limits_0^{\infty} a(\eta)\,F(\eta, \mu)\exp\Big(-\frac{x}{\eta}\Big)\,d\eta, \qquad x = \rho - R,$$

$$Y_2^{(1)}(\rho, \mu) = k_1\int\limits_0^{\infty}\exp\Big(-\frac{x}{\eta}\Big)\delta(\eta - \mu)\,d\eta,$$

$$a(\eta) = \eta(\eta - Q_1)\exp{(-\eta^2)}\,X(-\eta)k_1/(2|\lambda^+(\eta)|^2), \qquad Y_1^{(1)}(R, \mu) = (\mu^2 + Q_2)k_1.$$

In the case considered, we have

$$Z(\eta, \mu) = -\frac{1}{2}\,a(\eta)\frac{\partial F}{\partial\mu} + \mu k_1\delta(\eta - \mu) - \frac{k_1}{2}\,\frac{\partial}{\partial\mu}\,\delta(\eta - \mu),$$

$$\int\limits_{-\infty}^{\infty} P\,\frac{1}{\eta - \mu}\,Z(\eta, \mu)\exp{(-\mu^2)}\,d\mu = \frac{1}{2}\,a(\eta) + k_1\exp{(-\eta^2)}\Big(\eta^2 - \frac{1}{2}\Big),$$

$$\int\limits_0^{\infty}\eta Z(\eta, \mu)\,\frac{d\eta}{\eta - \mu} = -\frac{1}{3}\,\mu k_1,$$

$$f(\mu) = -2U_z^{(2)}\Big|_S + a(\mu)\exp{(\mu^2)}/2 + \mu^3 k_1,$$

$$U_z^{(2)}\Big|_S = -\frac{k_1}{2}\Big(Q_3 + \frac{1}{2\sqrt{\pi}}\int\limits_0^{\infty}\frac{t^2(t - Q_1)}{|\lambda^+(t)|^2}\,\exp{(-t^2)}\,dt\Big).$$

Hence, allowing for (1.1) and (2.13), we obtain

$$U_z^{(2)}\Big|_S = (Q_3 + Q_1Q_2)k_1/4, \qquad U_z\Big|_S = U_z^{(1)}\Big|_S + R^{-1}U_z^{(2)}\Big|_S = (0.38332 - 0.13339R^{-1})k_1$$

or, converting to dimensional quantities,

$$U_z\Big|_S = 1.149\,95\nu(1 - 0.589495\mathrm{Kn})\,\frac{1}{T_S}\,\frac{\partial T}{\partial z}\Big|_S.$$

Therefore, $\beta_{R\|} = 0.589495$.

Conclusions. In this work, creep velocities of a temperature-inhomogeneous rarefied gas along a solid cylindrical surface are found by soling the kinetic Boltzmann equation in the Knudsen layer with the collisions operator in the form of an operator of the ellipsoidal statistical model. The dependences of thermal creep coefficients on the curvature radius obtained in the approximation linear in terms of the Knudsen number have the same form as in [12]. In the case of a longitudinal flow past a cylindrical surface, the expressions obtained for the thermal creep velocity (3.2) coincide with the respective expressions in [12].

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